INVESTIGATION OF NEW WAYS TO MEASURE GAS DENSITY WITH AN ELECTRON BEAM METHOD

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Wide use is now made of the method of measuring the density of a rarefied gas from the intensity of radiation excited in the medium investigated by an electron beam [1, 2]. In this method, for a sufficiently narrow electron beam at the point x located on the beam axis, the density $\rho(x)$ is related to the intensity of excited radiation $I(x)$ by the simple equation $I(x) = Ai(x)\rho(x)$ ($i(x)$ is the beam current at the point x, and A is a constant). As the gas density increases there is substantial attenuation of the electron beam intensity, and the current $i(x)$ decreases with increase of x. Under these conditions the above relation cannot be used directly, since the value of $i(x)$ at points located in the volume investigated is unknown. References [3, 4] proposed methods of determining $\rho(x)$ from $I(x)$ under conditions of electron beam attenuation; in [3] it was posed to measure the density only at points x lying in the flow symmetry plane, and in [4] to determine the density profile $\rho(x)$ from the profile of radiative intensity at any point $x \in [x_0, x_k]$ located along the electron beam (x_0 is the entry point to the test region, and x_k is the exit point). In [4] the problem of determining $\rho(x)$ was reduced to seeking a solution of an integral equation, and an iterative algorithm was suggested for the solution. In contrast with [4] the present paper considers an approach that allows an exact solution of the equation presented in [4], and a detailed analysis of the errors in recovering $\rho(x)$ from $I(x)$, as a function of the level of errors of the directly measured quantities.

1. As was done in [3, 4] we shall use the widely applied model of exponential attenuation of the electron beam current with increase of x. Here at a point x lying on the beam axis we have the relation

$$i(x) = i_0 \exp\left[-\mu \int_0^x \rho(x_1)\,dx_1\right],$$

where i_0 is the current at the entrance to the test volume, i.e., at the point x_0 (for convenience we shall take $x_0 = 0$ as the origin of the variable x); and μ is the cross section for current attenuation.

In accordance with [4] the profile $I(x)$ and the profile $\rho(x)$ are related by the equation

$$a\rho(x)\exp\left[-\mu \int_0^x \rho(x_1)\,dx_1\right] = I(x) \quad (a = Ai_0). \tag{1.1}$$

We introduce the notation

$$z(x) = \int_0^x \rho(x_1)\,dx_1, \tag{1.2}$$

equivalent to the relation

$$\rho(x) = dz(x)/dx. \tag{1.3}$$

From Eq. (1.1), using Eqs. (1.2) and (1.3), we obtain the differential equation

$$a\exp[-\mu z(x)]\frac{dz(x)}{dx} = I(x). \tag{1.4}$$

Solving this by the method of separation of variables, we have

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$$a\int_0^{z(x)} \exp(-\mu z_1)\,dz_1 = \int_0^x I(x_1)\,dx_1. \tag{1.5}$$

After evaluating the integral on the left of Eq. (1.5) and carrying out a transformation, we find a solution of Eq. (1.4) in the form

$$z(x) = -\frac{1}{\mu}\ln\left[1-\frac{\mu}{a}\int_0^x I(x_1)\,dx_1\right]. \tag{1.6}$$

Taking into account that z(x) is the integral in Eq. (1.2) of ρ(x), as a result of differentiating Eq. (1.6) we obtain the final expression for the gas density

$$\rho(x) = \frac{I(x)}{a-\mu\int_0^x I(x_1)\,dx_1}, \tag{1.7}$$

where we have used the fact that $\frac{d}{dx}\int_0^x I(x_1)\,dx_1 = I(x)$.

In the special case of negligibly small decrease of the electron beam current (μ = 0), Eq. (1.7) reduces to the relation I(x) = aρ(x) used in [1, 2].

In order to calculate ρ(x) from Eq. (1.7) (for values of a and μ known as a result of calibration measurements) the single laborious operation is to find the integral of the profile I(x). The structure of Eq. (1.7) is such that when one uses an integrating element in the measuring system [5] one can automate the process of determining the density with computation of ρ(x) in the real time.

2. The quantities a, μ and I(x) appearing in Eq. (1.7) are usually measured with errors in the actual conditions of the experiment, i.e., instead of them we have, respectively,

$$\tilde{a} = a + \varepsilon_a,\ \tilde{\mu} = \mu + \varepsilon_\mu,\ \tilde{I}(x) = I(x) + \varepsilon_I(x). \tag{2.1}$$

Here ε_a, ε_μ are random deviations for the exact values; $\varepsilon_I(x)$ are random (noise) distortions of the function I(x). Substituting Eq. (2.1) into Eq. (1.7), we obtain instead of ρ(x) the distorted density profile

$$\tilde{\rho} = \rho(x) + \varepsilon_\rho(x) = \frac{I(x)+\varepsilon_I(x)}{g(x)+\Delta(x)}, \tag{2.2}$$

$$g(x) = a - \mu\int_0^x I(x_1)\,dx_1,$$

$$\Delta(x) = \varepsilon_a - \mu\int_0^x \varepsilon_I(x_1)\,dx_1 - \varepsilon_\mu\int_0^x [I(x_1)+\varepsilon_I(x_1)]\,dx_1.$$

Expanding the right side of Eq. (2.2) in a power series in the parameter $\bar{\Delta}(x) = \Delta(x)/g(x)$, we find

$$\rho(x) + \varepsilon_\rho(x) = \frac{I(x)+\varepsilon_I(x)}{g(x)}\left[1-\bar{\Delta}(x)+\bar{\Delta}^2(x)-\ldots\right]. \tag{2.3}$$

In practice we are interested in the case where the perturbing random factors ε_a, ε_μ and $\varepsilon_I(x)$ are quite small. In this situation in the expansion (2.3) we can limit attention to terms that are linear in the measurement errors ε_a, ε_μ and $\varepsilon_I(x)$. In this limited framework the error in determining ρ(x) can be represented in the form

$$\varepsilon_\rho(x) = \frac{1}{g(x)}\left\{\varepsilon_I(x) - I(x)\left[\varepsilon_a - \mu\int_0^x \varepsilon_I(x_1)\,dx_1 - \varepsilon_\mu\int_0^x I(x_1)\,dx_1\right]\right\}. \tag{2.4}$$

We can calculate the variance $\sigma_\rho^2(x)$ of the error $\varepsilon_\rho(x)$. If we assume that the noise in measuring $\varepsilon_I(x)$ is uncorrelated ("white noise"), then the variance of the integral

$$v(x) = \int_0^x \varepsilon_I(x_1)\,dx_1$$

can be written, in accordance with [6], in the form

$$\sigma_v^2(x) = 2x\sigma_I^2, \tag{2.5}$$

where σ_I^2 is the variance of the random function $\varepsilon_I(x)$ at each point of measurement (it is assumed that the measurement noise does not depend on x).

Since the measurement of a, μ and I(x) is usually conducted independently, the random characteristics ε_a, ε_μ and $\varepsilon_I(x)$ can be considered as statistically independent. Thus, by calculating $\sigma_\rho^2(x)$ as the variance of the sum of independent random quantities [7] and taking account of Eq. (2.5), we have

$$\sigma_\rho^2(x) = \frac{1}{g^2(x)}\left\{\sigma_I^2\left[1 + 2xI^2(x)\mu^2\right] + I^2(x)\left[\sigma_a^2 + \sigma_\mu^2\left(\int_0^x I(x_1)\,dx_1\right)^2\right]\right\}, \tag{2.6}$$

where σ_a^2 is the variance of the random function $\tilde{a}$; σ_μ^2 is the variance of the random function $\tilde{\mu}$.

3. Above we presented formulas for computing ρ(x) using Eq. (1.7) and the variance of the error $\sigma_\rho^2(x)$ from Eq. (2.6), in which we postulated a continuous method of recording the signal I(x). But if, because of various technical difficulties, I(x) can be recorded only at a finite set of points $\{x_n\}$, n = 1, ..., N, then, instead of the integral in the denominator of Eq. (1.7), we must use the quadrature formula [8]

$$\int_0^x I(x_1)\,dx_1 \approx \sum_{i=1}^{n} C_i^{(n)} I(x_i),$$

where x_i and $C_i^{(n)}$ are, respectively, the nodes and coefficients of the quadrature sum, and $n \le N$. It can be shown that in this case the variance $\sigma_\rho^2(x_n)$ at the point x_n is determined by the expression

$$\sigma_\rho^2(x_n) = \frac{1}{\tilde{g}^2(x_n)}\left\{\sigma_I^2\left[1 + \mu^2 I^2(x_n)\sum_{i=1}^{n}\left(C_i^{(n)}\right)^2\right] + I^2(x_n)\left[\sigma_a^2 + \sigma_\mu^2\left(\sum_{i=1}^{n} C_i^{(n)} I(x_i)\right)^2\right]\right\}, \tag{3.1}$$

and here

$$\rho(x_n) = \frac{I(x_n)}{a - \mu\sum_{i=1}^{n} C_i^{(n)} I(x_i)}. \tag{3.2}$$

It should be noted that replacement of the integral in Eq. (1.7) by the quadrature sum leads to an additional systematic error due to the discrete arithmetic. But if to calculate this distortion we use the expression

$$\xi(x) = \frac{\int_0^x I(x_1)\,dx_1 - \sum_{i=1}^{n} C_i^{(n)} I(x_i)}{\int_0^x I(x_1)\,dx_1},$$

then replacement of the integral in Eq. (1.7) by the quadrature sum is equivalent to using $\mu_1 = \mu/(1 - \xi(x))$ instead of μ. Thus, we can allow for the error of using discrete arithmetic by replacing the factor μ by the factor μ_1 in Eqs. (3.1) and (3.2) (if ξ(x) is known).

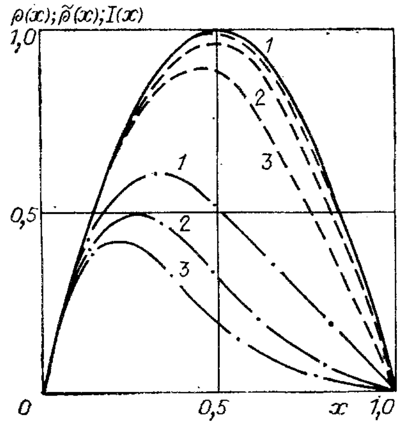

Fig. 1

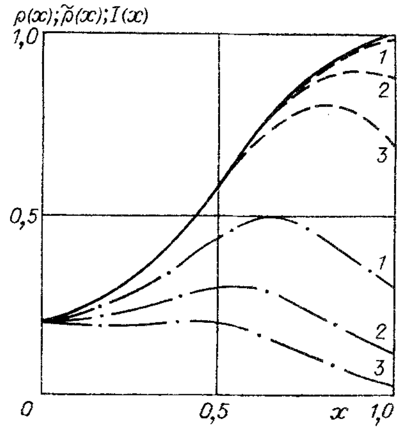

Fig. 2

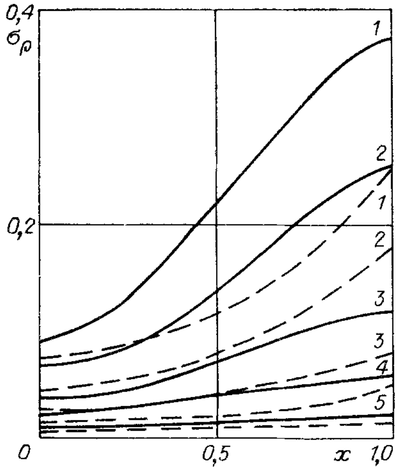

Fig. 3

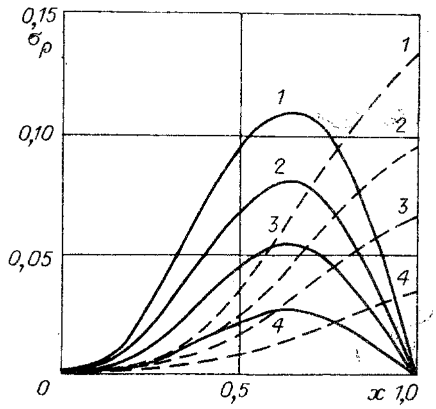

Fig. 4

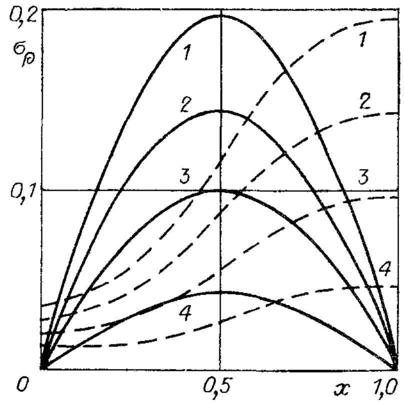

Fig. 5

4. To illustrate the possibilities of recovering $\rho(x)$ for the $I(x)$ profile we made test calculations for model functions. As original functions $\rho(x)$ we chose:

a parabolic profile

$$\rho(x) = 4x(1 - x); \tag{4.1}$$

a profile of shock wave type [9]

$$\rho(x) = \frac{1}{2}(\rho_1 + \rho_2) + \frac{1}{2}(\rho_1 - \rho_2)\,\mathrm{th}\left(\frac{2x - \Delta}{\Delta}\right) \tag{4.2}$$

with $\rho_1 = 0.2$, $\rho_2 = 1$, $\Delta = 1$.

For certain given values of the parameters a and μ, in accordance with Eq. (1.1) we calculated the intensity profile I(x), which was then summed with the computer-generated pseudorandom function $\varepsilon_I(x)$, having variance σ_I^2. From the function $\tilde{I}(x)$ thus obtained the density profile $\tilde{\rho}(x)$ was recovered. The results of this recovery are shown in Figs. 1 and 2 (for models (4.1) and (4.2), respectively), where the solid lines denote the exact function $\rho(x)$, and the broken lines denote the function I(x) with μ = 1, 2 and 3 (lines 1-3 and the values of μ are given in reduced units). By comparing the solid and broken lines we can see that in the model calculations we considered cases where the electron beam attenuation factor is appreciable. In the calculations we took a = 1.

The broken lines in Figs. 1 and 2 denote the recovered functions $\tilde{\rho}(x)$ also for μ = 1, 2 and 3. The noise σ_I was taken to be 1% of the maximum value of I(x).

Figures 3-5 show graphs of the dependence of the recovery error σ_p (for μ = 1), due to different levels of noise, error in the coefficients a and μ (the solid lines are the model (4.1), and the broken lines are the model (4.2)). The lines 1-5 on Fig. 3 refer to σ_I = 15; 10; 5; 3; and 1% of the $\max_x$ I(x), on Fig. 4 the lines 1-4 are for σ_μ = 20; 15; and 5%, and on Fig. 5 the lines 1-4 are for σ_a = 20; 15; 10; and 5%. The values of σ_I for the graphs of Figs. 4 and 5 were taken as 1%.

Analysis of the results of the model calculations shows that this method of recovering $\rho(x)$ has quite high stability towards errors in the original data.

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